CBSE 2026 EXAMINATIONS
XII MATHEMATICS (041)
Series PQSR2 ♦ Q.P. Code 65/2/1; 65/2/2; 65/2/3

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (**4 marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **03 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

☑ Q.P. Code - 65/2/1

SECTION A

This section has 20 multiple choice questions of 1 mark each.

01. A relation R on set $A = \{1,2,3\}$ defined as $R = \{(1,1),(2,2),(1,2)\}$ is

(a) Reflexive only (b) Reflexive and Transitive
(c) Symmetric and Transitive (d) Transitive only

Sol. (d) Transitive only.
As $(3,3) \notin R$. So, R can't be reflexive. Also $(1,2) \in R$ but $(2,1) \notin R$, so R can't be symmetric.

02. If A and B are square matrices of same order, then which of the following statements is/are always true?

(i) $(A+B)(A-B) = A^2 - B^2$
(ii) $AB = BA$
(iii) $(A+B)^2 = A^2 + AB + BA + B^2$
(iv) $AB = O \Rightarrow A = O$ or $B = O$

(a) Only (i) and (iii) (b) Only (ii) and (iii)
(c) Only (iii) (d) Only (iii) and (iv)

Sol. (c) Since $(A+B)^2 = (A+B)(A+B) = AA + AB + BA + BB = A^2 + AB + BA + B^2$.

03. If $A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $3a + b + c$ is

(a) 2 (b) 6 (c) 4 (d) 0

Sol. (a) $\because a_{ij} = a_{ji}$ for a symmetric matrix.

$\therefore a_{12} = a_{21} \Rightarrow a = -1$, $a_{13} = a_{31} \Rightarrow b = 0$, $a_{23} = a_{32} \Rightarrow c = 5$

Hence, $3a + b + c = 3(-1) + 0 + 5 = 2$.

04. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ and $A + A' = I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Sol. (c) $A + A' = I$ i.e., $\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} + \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By the def. of equality of matrices, we get $2\cos x = 1 \Rightarrow \cos x = \frac{1}{2}$

$\therefore x = \frac{\pi}{3}$.

05. For a square matrix A, $(3A)^{-1} =$

(a) $3A^{-1}$ (b) $9A^{-1}$ (c) $\frac{1}{3}A^{-1}$ (d) $\frac{1}{9}A^{-1}$

Sol. (c) Since $(kA)^{-1} = \frac{1}{k}A^{-1}; k \neq 0 \quad \therefore (3A)^{-1} = \frac{1}{3}A^{-1}$.

06. If $\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = -86$, then the sum of all possible values of a is

(a) 4 (b) 5 (c) –4 (d) 9

Sol. (c) Expanding the det. along 3rd row given in LHS, we get $0 - 4(1+10) + 2a(-a-4) = -86$

$\Rightarrow -2a^2 - 8a - 44 + 86 = 0$

$\Rightarrow a^2 + 4a - 21 = 0$

This is a quadratic equation, means we shall get two values of a (i.e., two roots of quadratic eq.).

So, sum of roots of the above quadratic equation = sum of all possible values of $a = -\frac{4}{1} = -4$.

07. If $e^{-x} + e^{-y} = 2$, then $\frac{dy}{dx}$ is

(a) e^{x-y} (b) e^{y-x} (c) $-e^{x-y}$ (d) $-e^{y-x}$

Sol. (d) On differentiating w.r.t. x both the sides, we get $e^{-x}(-1) + e^{-y}\left(-\frac{dy}{dx}\right) = 0$

$\Rightarrow -\left[e^{-x} + e^{-y}\frac{dy}{dx}\right] = 0$

$\Rightarrow e^{-y}\frac{dy}{dx} = -e^{-x}$

$\Rightarrow \frac{dy}{dx} = -e^{-x} \times e^{y} = -e^{y-x}$.

08. For $f(x) = x + \frac{1}{x} (x \neq 0)$

(a) local maximum value is 2
(b) local minimum value is –2
(c) local maximum value is –2
(d) local minimum value < local maximum value

Sol. (c) $f(x)=x+\frac{1}{x}$

$\Rightarrow f'(x)=1-\frac{1}{x^2}$ and $f''(x)=\frac{2}{x^3}$

For critical points, we have $f'(x)=1-\frac{1}{x^2}=0$

$\Rightarrow 1=\frac{1}{x^2}$

$\Rightarrow x^2=1$

$\therefore x=\pm 1$.

As $f''(1)=2>0$, $f''(-1)=-2<0$.

So, $x=1$ and $x=-1$ are the points of local minima and maxima respectively.

Hence, the local maximum value of function is, $f(-1)=-1+\frac{1}{-1}=-2$.

Note that, the local minimum value of function is, $f(1)=1+\frac{1}{1}=2$.

09. If $\int_0^{2a}\frac{1}{1+4x^2}dx=\frac{\pi}{6}$, then the value of a is

(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

Sol. (a) $\int_0^{2a}\frac{1}{1+4x^2}dx=\frac{\pi}{6}$

$\Rightarrow \frac{1}{2}\left[\tan^{-1}2x\right]_0^{2a}=\frac{\pi}{6}$

$\Rightarrow \tan^{-1}2\times 2a-\tan^{-1}0=\frac{\pi}{3}$

$\Rightarrow \tan^{-1}4a-0=\frac{\pi}{3}$

$\Rightarrow 4a=\tan\frac{\pi}{3}=\sqrt{3}$

$\therefore a=\frac{\sqrt{3}}{4}$.

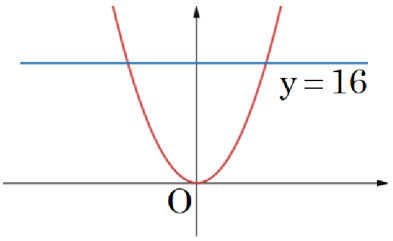
10. Which of the following expressions will give the area of region bounded by the curve $y=x^2$ and line $y=16$?

(a) $\int_0^4 x^2dx$ (b) $2\int_0^4 x^2dx$ (c) $\int_0^{16}\sqrt{y}\,dy$ (d) $2\int_0^{16}\sqrt{y}\,dy$

Sol. (d) Note that the curve $y=x^2$ is symmetrical about y axis.

Also $y=x^2$ implies, $\pm\sqrt{y}=x$.

So, required area $=2\int_0^{16}x\,dy=2\int_0^{16}\sqrt{y}\,dy$.



11. The general solution of the differential equation $\frac{dy}{dx}=\frac{\sqrt{y}}{\sqrt{x}}$ is

(a) $\log\sqrt{y}=\log\sqrt{x}+C$ (b) $\sqrt{y}+\sqrt{x}=C$

(c) $\sqrt{y}-\sqrt{x}=C$ (d) $\log\sqrt{y}+\log\sqrt{x}=C$

Sol. (c) Given $\frac{dy}{dx}=\frac{\sqrt{y}}{\sqrt{x}} \Rightarrow \int\frac{dy}{\sqrt{y}}=\int\frac{dx}{\sqrt{x}}$

$\Rightarrow \int\frac{dy}{2\sqrt{y}}=\int\frac{dx}{2\sqrt{x}}$

$\Rightarrow \sqrt{y}=\sqrt{x}+C$

$\therefore \sqrt{y}-\sqrt{x}=C.$

12. The integrating factor of the differential equation $2x\frac{dy}{dx}-y=3$ is

(a) $\sqrt{x}$ (b) $\frac{1}{\sqrt{x}}$ (c) e^{x} (d) e^{-x}

Sol. (b) Rewriting the D.E., $\frac{dy}{dx}+\left(\frac{-1}{2x}\right)y=\frac{3}{2x}$

On comparing with $\frac{dy}{dx}+P(x)y=Q(x)$, we get $P(x)=\frac{-1}{2x}$.

So, I.F. $=e^{\int\frac{-1}{2x}dx}=e^{-\frac{1}{2}\log x}=e^{\log x^{-1/2}}=x^{-1/2}=\frac{1}{\sqrt{x}}$.

13. If $|\vec{a}|=5$ and $-2\le\lambda\le1$, then the sum of greatest and the smallest value of $|\lambda\vec{a}|$ is

(a) –5 (b) 5 (c) 10 (d) 15

Sol. (c) Since $-2\le\lambda\le1$ gives $0\le|\lambda|\le2$

$\Rightarrow 0\times|\vec{a}|\le|\vec{a}||\lambda|\le2\times|\vec{a}|$

$\Rightarrow 0\le|\lambda\vec{a}|\le10$ [$\because|\vec{a}|=5$

So, greatest value of $|\lambda\vec{a}|$ is 10 and the smallest value is 0.

The required sum is $10+0=10$.

14. Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z axis is

(a) $\hat{i}+2\sqrt{2}\,\hat{j}$ (b) $3\hat{k}$ (c) $\frac{3\sqrt{2}}{2}\hat{i}+\frac{3\sqrt{2}}{2}\hat{j}$ (d) $\sqrt{3}\,\hat{i}+\sqrt{3}\,\hat{j}+\sqrt{3}\,\hat{k}$

Sol. (c) Suppose $\vec{r}$ be the required vector such that $|\vec{r}|=3$. Clearly, $\vec{r}=|\vec{r}|\hat{r}=3\hat{r}$.

Let the angles made by the vector $\vec{r}$ with x and y axes be θ. That is, $\alpha=\beta=\theta$, also $\gamma=\frac{\pi}{2}$.

Using $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$, we get $\cos^2\theta+\cos^2\theta+\cos^2\frac{\pi}{2}=1$

$\Rightarrow 2\cos^2\theta+0=1$

$\Rightarrow \cos^2\theta=\frac{1}{2} \Rightarrow \cos\theta=\frac{1}{\sqrt{2}}$

Hence, the required vector $\vec{r}=3\hat{r}=3\left(\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}+0\hat{k}\right)=3\left(\frac{\sqrt{2}}{2}\hat{i}+\frac{\sqrt{2}}{2}\hat{j}\right)=\frac{3\sqrt{2}}{2}\hat{i}+\frac{3\sqrt{2}}{2}\hat{j}$.

Note, a unit vector can be written as $\cos\alpha\,\hat{i}+\cos\beta\,\hat{j}+\cos\gamma\,\hat{k}$.

15. Direction cosines of the line given by the equations : $\frac{2x-1}{4}=\frac{1-y}{3}=\frac{-z}{6}$ are

(a) $2,-3,-6$ (b) $\frac{2}{7},\frac{-3}{7},\frac{-6}{7}$ (c) $\frac{2}{7},\frac{-3}{7},\frac{6}{7}$ (d) $\frac{4}{\sqrt{61}},\frac{-3}{\sqrt{61}},\frac{-6}{\sqrt{61}}$

Sol. (b) Rewriting the line, we get $\frac{x-\frac{1}{2}}{2}=\frac{y-1}{-3}=\frac{z}{-6}$.

So, the d.r.'s of the line are 2, –3, –6.

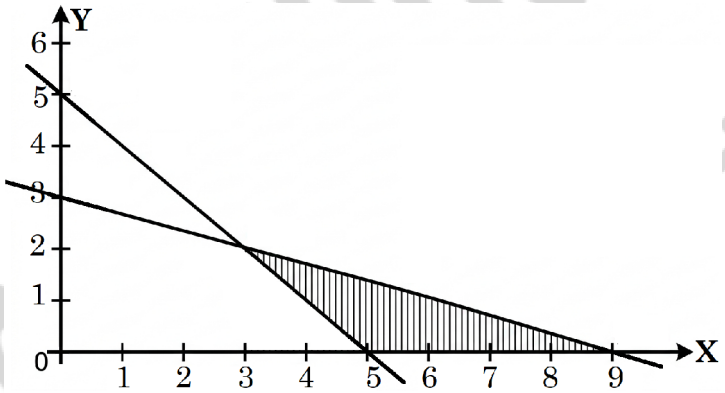
Hence, its d.c.'s are given by $\frac{2}{\sqrt{2^2+(-3)^2+(-6)^2}},\frac{-3}{\sqrt{4+9+36}},\frac{-6}{\sqrt{49}}$ i.e., $\frac{2}{7},\frac{-3}{7},\frac{-6}{7}$.

16. In a linear programming problem, the linear function which has to be maximized or minimized is called

(a) a feasible function (b) an objective function
(c) an optimal function (d) a constraint

Sol. (b) an objective function.

17. For the feasible region shown below, the non-trivial constraints of the linear programming problem are



(a) $x+y\le 5,\ x+3y\le 9$ (b) $x+y\le 5,\ x+3y\ge 9$
(c) $x+y\ge 5,\ x+3y\le 9$ (d) $x+y\ge 5,\ 3x+y\le 9$

Sol. (c) Note that, one of the line shown passes through (5, 0) and (0, 5); while the other line passes through (9, 0) and (0, 3).

Their corresponding equations are $\frac{x}{5}+\frac{y}{5}=1,\ \frac{x}{9}+\frac{y}{3}=1$ i.e., $x+y=5,\ x+3y=9$.

Now as per the shown feasible region, we must have the constraints as $x+y\ge 5,\ x+3y\le 9$.

18. For two events A and B such that $P(A)\ne 0$ and $P(B)\ne 1$, $P(A'\,|\,B')=$

(a) $1-P(A\,|\,B)$ (b) $1-P(A'\,|\,B)$ (c) $\frac{1-P(A\cap B)}{P(B')}$ (d) $\frac{1-P(A\cup B)}{P(B')}$

Sol. (d) $P(A'\,|\,B')=\frac{P(A'\cap B')}{P(B')}=\frac{P(A\cup B)'}{P(B')}=\frac{1-P(A\cup B)}{P(B')}$.

Direction : *Question numbers* ***19*** *and* ***20*** *are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. For two vectors $\vec{a}$ and $\vec{b}$,

Assertion (A) : $\left|\vec{a}\times\vec{b}\right|^2+(\vec{a}.\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$.

Reason (R) : $\left|\vec{a}\times\vec{b}\right|=(\vec{a}.\vec{b})\tan\theta,\left(\theta\neq\frac{\pi}{2}\right)$.

Sol. (b) $\because \left|\vec{a}\times\vec{b}\right|^2+(\vec{a}.\vec{b})^2=|\vec{a}|^2|\vec{b}|^2\sin^2\theta+|\vec{a}|^2|\vec{b}|^2\cos^2\theta=|\vec{a}|^2|\vec{b}|^2\left(\sin^2\theta+\cos^2\theta\right)=|\vec{a}|^2|\vec{b}|^2\times1=|\vec{a}|^2|\vec{b}|^2$

Also $\left|\vec{a}\times\vec{b}\right|=|\vec{a}||\vec{b}|\sin\theta=|\vec{a}||\vec{b}|\cos\theta\times\frac{\sin\theta}{\cos\theta}=|\vec{a}||\vec{b}|\cos\theta\times\tan\theta=(\vec{a}.\vec{b})\tan\theta;\left(\theta\neq\frac{\pi}{2}\right)$.

$\therefore$ Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).

REMARK It is not mention that θ is the angle between $\vec{a}$ and $\vec{b}$.

20. **Assertion (A) :** A line can have direction cosines $<1, 1, 1>$.

Reason (R) : $\cos\theta=1$ is possible for $\theta=0$.

Sol. (d) Since the given set of d.c.'s in Assertion does not satisfy $l^2+m^2+n^2=1$.
$\therefore$ Assertion (A) is false but Reason (R) is true.

SECTION B

This section has 5 Very Short Answer questions of 2 marks each.

21. (a) Check whether $f:\mathbb{R}-\{3\}\to\mathbb{R}$ defined as $f(x)=\frac{x-2}{x-3}$ is onto or not

OR

(b) Check whether $f:Z\times Z\to Z\times Z$ (where Z is the set of integers) defined as $f(x,y)=(2y,3x)$ is injective or not.

Sol. (a) Let $y=f(x),\ y\in\mathbb{R}$.

Then $y=\frac{x-2}{x-3}$

$\Rightarrow xy-3y=x-2$

$\Rightarrow xy-x=3y-2$

$\Rightarrow x(y-1)=3y-2$

$\Rightarrow x=\frac{3y-2}{y-1}$

For x to be a real number, the denominator must not be zero. Therefore $y\neq1$.

This means that for $y=1$, there is no real value of x such that $f(x)=1$.

That is, range of $f(x)$ is $\mathbb{R}-\{1\}$ which is not same as the given codomain ($\mathbb{R}$) of $f(x)$.

Hence, $f(x)$ is not onto.

OR

(b) Let (x_1, y_1) and (x_2, y_2) be two elements in the domain $Z\times Z$ s.t. $f(x_1, y_1)=f(x_2, y_2)$.

So, $(2y_1, 3x_1)=(2y_2, 3x_2)$

For two ordered pairs to be equal, their corresponding components must be equal.

That is, $2y_1=2y_2$ and $3x_1=3x_2$

$\Rightarrow y_1=y_2$ and $x_1=x_2$.

Then we can write $(x_1, y_1) = (x_2, y_2)$

As $f(x_1, y_1) = f(x_2, y_2)$ leads to $(x_1, y_1) = (x_2, y_2)$, the function is injective (one-one).

22. If $x = a\sin^3 t$, $y = b\cos^3 t$, then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

Sol. $x = a\sin^3 t$, $y = b\cos^3 t$

On differentiating w.r.t. t, we get $\frac{dx}{dt} = 3a\sin^2 t\cos t$, $\frac{dy}{dt} = 3b\cos^2 t(-\sin t)$

$\because \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\therefore \frac{dy}{dx} = \frac{3b\cos^2 t(-\sin t)}{3a\sin^2 t\cos t} = -\frac{b}{a}\cot t$

Hence $\frac{dy}{dx}\left(\text{at } t = \frac{\pi}{4}\right) = -\frac{b}{a}\cot\frac{\pi}{4} = -\frac{b}{a}\times 1 = -\frac{b}{a}$.

23. (a) Find the absolute maximum value of $f(x) = \cos x + \sin^2 x$, $x \in [0, \pi]$.

OR

(b) If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius.

Sol. (a) $f(x) = \cos x + \sin^2 x$, $x \in [0, \pi]$

$\Rightarrow f'(x) = -\sin x + 2\sin x\cos x$

For $f'(x) = 0$, $-\sin x(1 - 2\cos x) = 0$

Either $\sin x = 0$ or $\cos x = \frac{1}{2}$

$\therefore x = 0, \frac{\pi}{3} \in [0, \pi]$.

Now $f(0) = \cos 0 + \sin^2 0 = 1$, $f\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} + \sin^2\frac{\pi}{3} = \frac{1}{2} + \frac{3}{4} = \frac{5}{4}$ and $f(\pi) = \cos\pi + \sin^2\pi = -1$.

Clearly, the absolute maximum value of $f(x)$ is $\frac{5}{4}$.

OR

(b) Given $\frac{dV}{dt} = k$, where k is a constant.

Then $\frac{d}{dt}\left(\frac{2}{3}\pi r^3\right) = k \Rightarrow 2\pi r^2\frac{dr}{dt} = k$

$\therefore \frac{dr}{dt} = \frac{k}{2\pi r^2}$ ...(i)

Now surface area, $A = 3\pi r^2$

$\Rightarrow \frac{dA}{dt} = 6\pi r \times \frac{dr}{dt}$

By (i), $\frac{dA}{dt} = 6\pi r \times \frac{k}{2\pi r^2} = \frac{3k}{r}$

$\Rightarrow \frac{dA}{dt} \propto \frac{3k}{r}$.

Therefore, the rate of change of surface area varies inversely as the radius.

24. If $\overrightarrow{AB}=\hat{j}+\hat{k}$ and $\overrightarrow{AC}=3\hat{i}-\hat{j}+4\hat{k}$ represent the two vectors along the sides AB and AC of ΔABC, prove that the median $\overrightarrow{AD}=\dfrac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$, where D is midpoint of BC.

Hence, find the length of median AD.

Sol. In ΔABC, let $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$, $\overrightarrow{OC}=\vec{c}$.

Then $\overrightarrow{AB}=\vec{b}-\vec{a}$, $\overrightarrow{AC}=\vec{c}-\vec{a}$.

Also D is the midpoint of BC, so $\overrightarrow{OD}=\dfrac{\vec{b}+\vec{c}}{2}$.

Now the median vector is $\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}=\dfrac{\vec{b}+\vec{c}}{2}-\vec{a}=\dfrac{\vec{b}+\vec{c}-2\vec{a}}{2}=\dfrac{(\vec{b}-\vec{a})+(\vec{c}-\vec{a})}{2}=\dfrac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$.

Hence, $\overrightarrow{AD}=\dfrac{(\hat{j}+\hat{k})+(3\hat{i}-\hat{j}+4\hat{k})}{2}=\dfrac{3\hat{i}+5\hat{k}}{2}=\dfrac{3\hat{i}}{2}+\dfrac{5\hat{k}}{2}$.

$\therefore$ Length of median AD is $\left|\overrightarrow{AD}\right|=\sqrt{\left(\dfrac{3}{2}\right)^2+0^2+\left(\dfrac{5}{2}\right)^2}=\sqrt{\dfrac{9}{4}+\dfrac{25}{4}}=\sqrt{\dfrac{9+25}{4}}=\dfrac{\sqrt{34}}{2}$ units.

25. Find the coordinates of the point on the line $\vec{r}=-\hat{j}+3\hat{k}+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ such that the sum of coordinates is 3.

Sol. On the given line, the coordinates of a random point are $(2\lambda,-2\lambda-1,\lambda+3)$.

Now taking the sum of the coordinates, we get $(2\lambda)+(-2\lambda-1)+(\lambda+3)=3$

$\Rightarrow \lambda=1$

$\therefore$ The required coordinates are (2, –3, 4).

SECTION C

There are 6 Short Answer questions in this section. Each is of 3 marks.

26. Find $\int\dfrac{x+2}{\sqrt{9x-x^2}}dx$.

Sol. Let $I=\int\dfrac{x+2}{\sqrt{9x-x^2}}dx$

$\Rightarrow I=-\dfrac{1}{2}\int\dfrac{-2x-4}{\sqrt{9x-x^2}}dx$

$\Rightarrow I=-\dfrac{1}{2}\int\dfrac{(9-2x)-13}{\sqrt{9x-x^2}}dx$

$\Rightarrow I=-\dfrac{1}{2}\int\dfrac{(9-2x)}{\sqrt{9x-x^2}}dx+\dfrac{1}{2}\int\dfrac{13}{\sqrt{9x-x^2}}dx$

$\Rightarrow I=-\dfrac{1}{2}\times 2\sqrt{9x-x^2}+\dfrac{13}{2}\int\dfrac{1}{\sqrt{\left(\dfrac{9}{2}\right)^2-\left(x-\dfrac{9}{2}\right)^2}}dx$ $\left[\because\int\dfrac{f'(x)}{\sqrt{f(x)}}dx=2\sqrt{f(x)}+C\right.$

$\Rightarrow I=-\sqrt{9x-x^2}+\dfrac{13}{2}\sin^{-1}\left(\dfrac{x-\dfrac{9}{2}}{\dfrac{9}{2}}\right)+C$

$\therefore I=-\sqrt{9x-x^2}+\dfrac{13}{2}\sin^{-1}\left(\dfrac{2x-9}{9}\right)+C$.

27. (a) Evaluate $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}}$.

OR

(b) Evaluate $\int_{\frac{-\pi}{6}}^{\frac{\pi}{2}} \left(\sin|x|+\cos|x|\right)dx$.

Sol. (a) Let $I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}}$

$$\Rightarrow I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\frac{\cos x}{\sin x}}}$$

$$\Rightarrow I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt{\sin x}\,dx}{\sqrt{\sin x}+\sqrt{\cos x}} \quad ...(i)$$

$$\Rightarrow I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt{\sin\left(\frac{\pi}{12}+\frac{5\pi}{12}-x\right)}\,dx}{\sqrt{\sin\left(\frac{\pi}{12}+\frac{5\pi}{12}-x\right)}+\sqrt{\cos\left(\frac{\pi}{12}+\frac{5\pi}{12}-x\right)}}$$

$$\Rightarrow I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}\,dx}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}$$

$$\Rightarrow I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt{\cos x}\,dx}{\sqrt{\cos x}+\sqrt{\sin x}} \quad ...(ii)$$

Adding (i) and (ii),we get $2I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} 1\,dx = [x]_{\pi/12}^{5\pi/12} = \frac{5\pi}{12}-\frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$

$\therefore I = \frac{\pi}{6}$.

OR

(b) Let $I = \int_{\frac{-\pi}{6}}^{\frac{\pi}{2}} \left(\sin|x|+\cos|x|\right)dx$

$$\Rightarrow I = \int_{\frac{-\pi}{6}}^{0} \left(\sin|x|+\cos|x|\right)dx + \int_{0}^{\frac{\pi}{2}} \left(\sin|x|+\cos|x|\right)dx$$

$$\Rightarrow I=\int_{\frac{-\pi}{6}}^{0}\left(\sin(-x)+\cos(-x)\right)dx+\int_{0}^{\frac{\pi}{2}}\left(\sin x+\cos x\right)dx$$

$$\Rightarrow I=\int_{\frac{-\pi}{6}}^{0}\left(-\sin x+\cos x\right)dx+\left[\sin x-\cos x\right]_{0}^{\pi/2}$$

$$\Rightarrow I=\left[\cos x+\sin x\right]_{-\pi/6}^{0}+\left[\sin\frac{\pi}{2}-\cos\frac{\pi}{2}\right]-\left[\sin 0-\cos 0\right]$$

$$\Rightarrow I=\left[\cos 0+\sin 0\right]-\left[\cos\left(\frac{-\pi}{6}\right)+\sin\left(\frac{-\pi}{6}\right)\right]+\left[1-0\right]-\left[0-1\right]$$

$$\Rightarrow I=1-\left[\frac{\sqrt{3}}{2}-\frac{1}{2}\right]+1+1$$

$$\Rightarrow I=3+\frac{1}{2}-\frac{\sqrt{3}}{2}$$

$$\therefore I=\frac{7-\sqrt{3}}{2}.$$

28. If $\frac{d}{dx}\left(F(x)\right)=\frac{1}{e^x+1}$, then find $F(x)$ given that $F(0)=\log\frac{1}{2}$.

Sol. As $\frac{d}{dx}\left(F(x)\right)=\frac{1}{e^x+1}$ gives $d\left(F(x)\right)=\frac{1}{e^x+1}dx$

On integrating both the sides, we get $\int d\left(F(x)\right)=\int\frac{1}{e^x+1}dx$

$$\Rightarrow F(x)=\int\frac{e^{-x}}{1+e^{-x}}dx$$ [Dividing by e^x in Nr and Dr both

$$\Rightarrow F(x)=-\int\frac{-e^{-x}}{1+e^{-x}}dx$$

$$\therefore F(x)=-\log\left|1+e^{-x}\right|+C \quad ...(i)$$ $\left[\because\int\frac{f'(x)}{f(x)}dx=\log\left|f(x)\right|+C\right.$

Since $F(0)=\log\frac{1}{2}$ so, $F(0)=-\log\left|1+e^{-0}\right|+C=-\log\left|1+1\right|+C$

$$\Rightarrow\log\frac{1}{2}=-\log 2+C$$

$$\Rightarrow\log\frac{1}{2}=\log\frac{1}{2}+C$$

$$\Rightarrow C=0$$

By (i), we get $F(x)=-\log\left|1+e^{-x}\right|$.

Also $F(x)=-\log\left|\frac{e^x+1}{e^x}\right|=-\left[\log\left|e^x+1\right|-\log\left|e^x\right|\right]=x\log e-\log\left|e^x+1\right|=x-\log\left|e^x+1\right|$.

29. (a) Solve the following differential equation $x\frac{dy}{dx} = y - x\sin^2\left(\frac{y}{x}\right)$, given that $y(1) = \frac{\pi}{6}$.

OR

(b) Find the general solution of the differential equation $y\log y\frac{dx}{dy} + x = \frac{2}{y}$.

Sol. (a) Rewriting the D.E., we get $\frac{dy}{dx} = \frac{y}{x} - \sin^2\left(\frac{y}{x}\right)$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

So, $v + x\frac{dv}{dx} = v - \sin^2 v$

$\Rightarrow x\frac{dv}{dx} = -\sin^2 v$

$\Rightarrow \int -\text{cosec}^2 v\, dv = \int \frac{dx}{x}$

$\Rightarrow \cot v = \log|x| + C$

$\Rightarrow \cot\frac{y}{x} = \log|x| + C$

It is given that $y(1) = \frac{\pi}{6}$ so, $\cot\frac{\pi}{6} = \log|1| + C$

$\Rightarrow \sqrt{3} = 0 + C$

$\Rightarrow C = \sqrt{3}$

Hence, the required solution is $\cot\frac{y}{x} = \log|x| + \sqrt{3}$.

OR

(b) Rewriting the D.E., we get $\frac{dx}{dy} + \frac{1}{y\log y}x = \frac{2}{y^2\log y}$

On comparing with $\frac{dx}{dy} + P(y)x = Q(y)$, we get $P(y) = \frac{1}{y\log y}$, $Q(y) = \frac{2}{y^2\log y}$.

So, I.F. $= e^{\int\frac{1}{y\log y}dy} = e^{\log\log y} = \log y$.

Therefore, the solution is given by $\log y \times x = \int \log y \times \frac{2}{y^2\log y}dy + C$

$\Rightarrow x\log y = \int\frac{2}{y^2}dy + C$

$\Rightarrow x\log y = -\frac{2}{y} + C$.

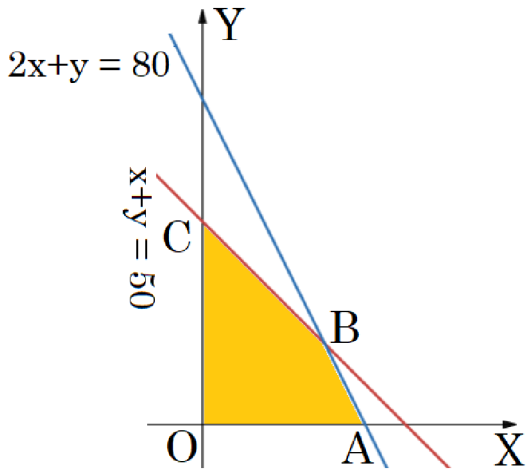
30. Solve the following linear programming problem graphically.

Maximize $Z = 10500x + 9000y$

Subject to constraints $x + y \le 50$, $2x + y \le 80$; $x, y \ge 0$.

Sol. Consider the graph shown.
The corner points are O(0, 0), A(40, 0), B(30, 20) and C(0, 50).
Therefore, $Z_O = 0$, $Z_A = 420000$, $Z_B = 495000$, $Z_C = 450000$.

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Hence, the maximum value of Z is 495000 at the corner point (30, 20).

31. (a) The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day due to high wind speed.



The sniper fired two shots on the target on a stormy day when wind speed was very high. Find the probability that

(i) target is hit.

(ii) atleast one shot misses the target.

OR

(b) Mother, Father and Son line up at random for a family picture. Let event E : Son on one end and F : Father in the middle. Find $P(E|F)$.

Sol. (a) Let the probability of hitting the target be 'p' and that of not hitting be 'q'.

$\therefore p = 3q$

Also using $p+q=1$, we get $3q+q=1 \Rightarrow q=\frac{1}{4}$, $p=\frac{3}{4}$.

The sniper fires two shots.

(i) 'Target is hit' means either only one shot hits or both shot hits.

$\therefore$ Required probability $= 1-q\times q = 1-\frac{1}{4}\times\frac{1}{4} = 1-\frac{1}{16} = \frac{15}{16}$. [$\because pp+pq+qp+qq=1$

(ii) Required probability $= q\times q + p\times q + q\times p = \frac{1}{4}\times\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4} = \frac{1+3+3}{16} = \frac{7}{16}$.

OR

(b) There are 3 people - Mother (M), Father (F), Son (S).

Total possible arrangements $= 3! = 6$.

We have to find $P(E|F)$, meaning probability that the Son is on one end given that the Father is in the middle.

When the Father in the middle, we have possible arrangements as MFS, SFM.

So, $P(F)=\frac{2}{6}$.

For $E\cap F$ i.e., Son on end and Father in the middle, the arrangements are SFM, MFS.

So, $P(E\cap F)=\frac{2}{6}$.

Hence, $P(E\,|\,F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{2}{6}}{\frac{2}{6}}$

$\therefore P(E\,|\,F)=1$.

SECTION D

There are 4 Long Answer questions in this section. Each is of 5 marks.

32. (a) If $P=\begin{bmatrix}1&-1&0\\2&3&4\\0&1&2\end{bmatrix}$ and $Q=\begin{bmatrix}2&2&-4\\-4&2&-4\\2&-1&5\end{bmatrix}$, find (QP) and hence solve the following system of equations using matrices : $x-y=3,\ 2x+3y+4z=17,\ y+2z=7$

OR

(b) Obtain the value of $\Delta=\begin{vmatrix}1+x&1&1\\1&1+y&1\\1&1&1+z\end{vmatrix}$ in terms of x, y and z. Further, if $\Delta=0$ and x, y, z are non-zero real numbers, prove that $x^{-1}+y^{-1}+z^{-1}=-1$.

Sol. (a) Here $P=\begin{bmatrix}1&-1&0\\2&3&4\\0&1&2\end{bmatrix}$ and $Q=\begin{bmatrix}2&2&-4\\-4&2&-4\\2&-1&5\end{bmatrix}$

$$\therefore QP=\begin{bmatrix}2&2&-4\\-4&2&-4\\2&-1&5\end{bmatrix}\begin{bmatrix}1&-1&0\\2&3&4\\0&1&2\end{bmatrix}$$

$$\Rightarrow QP=\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix}.$$

That is, $QP=6I$ …(i)

Consider the given systems of equations : $x-y=3,\ 2x+3y+4z=17$ and $y+2z=7$.

These equations can be expressed as : $AX=D$ where $A=\begin{bmatrix}1&-1&0\\2&3&4\\0&1&2\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $D=\begin{bmatrix}3\\17\\7\end{bmatrix}$

Therefore, $X=A^{-1}D=(P)^{-1}D=(P^{-1})D=\frac{1}{6}QD$

$\left[\begin{array}{l}\text{By (i), } QP=6I\Rightarrow P^{-1}=\frac{1}{6}Q\\ \because A=P\quad\therefore A^{-1}=P^{-1}=\frac{1}{6}Q\end{array}\right.$

So, $X=\frac{1}{6}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}\begin{bmatrix}3\\17\\7\end{bmatrix}$

$\Rightarrow X=\frac{1}{6}\begin{bmatrix}12\\-6\\24\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-1\\4\end{bmatrix}$

$\therefore$ By equality of matrices, we get : $x=2,\ y=-1,\ z=4$.

OR

(b) Expanding the given determinant $\Delta=\begin{vmatrix}1+x & 1 & 1\\ 1 & 1+y & 1\\ 1 & 1 & 1+z\end{vmatrix}$ along 1^{st} row, we get

$\Delta=(1+x)\left[(1+y)(1+z)-1\right]-1\left[(1+z)-1\right]+1\left[1-(1+y)\right]$

$\Rightarrow \Delta=(1+x)\left[1+y+z+yz-1\right]-z-y$

$\Rightarrow \Delta=(1+x)\left[y+z+yz\right]-z-y$

$\Rightarrow \Delta=y+z+yz+xy+xz+xyz-z-y$

$\Rightarrow \Delta=xy+yz+zx+xyz$.

Taking $\Delta=0$, we get $xy+yz+zx+xyz=0$

Dividing both sides by xyz, we get $\frac{xy+yz+zx+xyz}{xyz}=\frac{0}{xyz}$ [$\because x\neq 0,\ y\neq 0,\ z\neq 0$

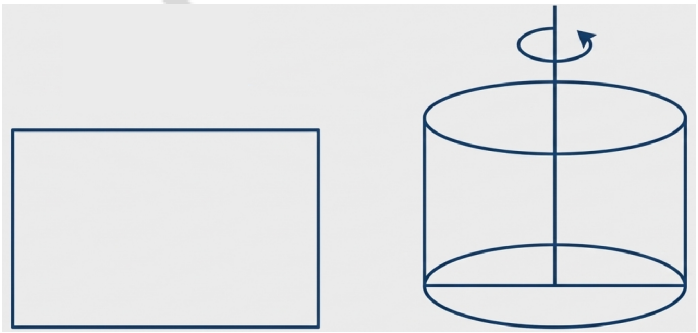
$\Rightarrow \frac{1}{z}+\frac{1}{x}+\frac{1}{y}+1=0$

$\therefore x^{-1}+y^{-1}+z^{-1}=-1$.

33. (a) Find the sub intervals in which $f(x)=\cot^{-1}(\sin x+\cos x),\ x\in(0,\ \pi)$ is increasing and decreasing.

OR

(b) A rectangle of perimeter 36 cm is revolved around one of its sides to sweep out a cylinder of maximum volume.



Find the dimensions of the rectangle.

Sol. (a) For $f(x) = \cot^{-1}(\sin x + \cos x),\ x \in (0,\ \pi)$

$$\Rightarrow f'(x) = -\frac{1}{1+(\sin x+\cos x)^2}\times(\cos x - \sin x)$$

$$\Rightarrow f'(x) = -\frac{(\cos x-\sin x)}{1+\sin^2 x+\cos^2 x+2\sin x\cos x} = -\frac{(\cos x-\sin x)}{2+\sin 2x} = \frac{(\sin x-\cos x)}{2+\sin 2x}$$

For $f'(x) = 0$, $\dfrac{(\sin x-\cos x)}{2+\sin 2x} = 0 \qquad \Rightarrow \sin x = \cos x$

$$\therefore x = \frac{\pi}{4} \in (0,\ \pi)$$

For $x \in \left(0,\ \dfrac{\pi}{4}\right]$, $f'(x) \le 0$ so, f is decreasing in this interval.

For $x \in \left[\dfrac{\pi}{4}, \pi\right)$, $f'(x) \ge 0$ so, f is increasing in this interval.

OR

(b) Suppose the rectangle is rotated about its length. Then the length L of rectangle will be height h of the cylinder thus generated. Also the width b of rectangle will be acting as the radius of the base of cylinder.

As perimeter of the rectangle $= 2(L+b) = 36 \qquad \Rightarrow L = 18 - b$

Now volume of the cylinder is, $V = \pi r^2 h$

$$\Rightarrow V = \pi b^2 L$$

$$\Rightarrow V = \pi b^2(18-b)$$

$$\Rightarrow V = \pi(18b^2 - b^3)$$

$$\Rightarrow \frac{dV}{db} = \pi(36b - 3b^2)$$

$$\Rightarrow \frac{d^2V}{db^2} = \pi(36-6b)$$

For local points of maxima and/or minima, $\dfrac{dV}{db} = \pi(36b-3b^2) = 0$

$$\Rightarrow 3\pi b(12-b) = 0$$

$\because b \ne 0 \ \therefore b = 12$ cm

$$\because \left.\frac{d^2V}{db^2}\right]_{\text{at } b=12 \text{ cm}} = \pi(36-72) = -36\pi < 0,$$ so V is maximum at $b = 12$ cm.

As $L = 18 - b$ so, $L = 18 - 12 = 6$ cm.

Hence, the dimensions of rectangle are 12 cm $\times$ 6 cm.

34. Find the domain of $g(x) = \cos^{-1}(x^2-1)$. Hence, find the value of x for which $g(x) = \dfrac{\pi}{3}$. Also, write the range of $\cos^{-1} x$ other than its principal branch.

Sol. For the function $g(x)$ to be defined, we must have $-1 \le (x^2-1) \le 1$

$$\Rightarrow -1+1 \le (x^2-1)+1 \le 1+1$$

$$\Rightarrow 0 \le x^2 \le 2$$

$$\therefore \text{Domain} = x \in \left[-\sqrt{2},\ \sqrt{2}\right].$$

When $g(x)=\frac{\pi}{3}$, we get $\cos^{-1}(x^2-1)=\frac{\pi}{3}$

$\Rightarrow \cos\cos^{-1}(x^2-1)=\cos\frac{\pi}{3}$

$\Rightarrow (x^2-1)=\frac{1}{2}$

$\Rightarrow x^2=\frac{1}{2}+1$

$\Rightarrow x^2=\frac{3}{2}$

$\Rightarrow x=\pm\sqrt{\frac{3}{2}}$

$\therefore x=\pm\frac{\sqrt{6}}{2}$.

Also, a range of $\cos^{-1}x$ other than its principal branch can be taken as $[-\pi, 0]$ or $[\pi, 2\pi]$ etc.

35. A line passing through the points A(1, 2, 3) and B(5, 8, 11) intersects the line $\vec{r}=4\hat{i}+\hat{j}+\lambda(5\hat{i}+2\hat{j}+\hat{k})$. Find the coordinates of the point of intersection. Hence, write the equation of a line passing through the point of intersection and perpendicular to both the lines.

Sol. The line passing through the points A(1, 2, 3) and B(5, 8, 11) is given by

$L_1:\frac{x-1}{5-1}=\frac{y-2}{8-2}=\frac{z-3}{11-3}$ i.e., $L_1:\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\mu$ (say)

Also the given line is $L_2:\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{1}=\lambda$.

Taking the coordinates of random points on the respective lines, as $P(2\mu+1,3\mu+2,4\mu+3)$ and $Q(5\lambda+4,2\lambda+1,\lambda)$.

When the lines intersect, the points P and Q must coincide.

Therefore, $2\mu+1=5\lambda+4$ ...(i), $3\mu+2=2\lambda+1$ ...(ii), $4\mu+3=\lambda$ ...(iii)

Solving (i) and (iii), we get $2\mu+1=5(4\mu+3)+4$

$\Rightarrow 18\mu=-18$

$\therefore \mu=-1,\ \lambda=-1$

Note that, LHS of (ii) $=3\mu+2=3(-1)+2=-1$, RHS of (ii) $=2\lambda+1=2(-1)+1=-1$.

As these values of $\mu=-1$, $\lambda=-1$ satisfy (ii), so the lines L_1 and L_2 intersect.

Hence, the point of intersection is (–1, –1, –1)

Let the required equation of line passing through (–1, –1, –1) be $L:\frac{x+1}{a}=\frac{y+1}{b}=\frac{z+1}{c}$.

Since $L\perp L_1$ and $L\perp L_2$ so, by using $a_1a_2+b_1b_2+c_1c_2=0$, we get

$2a+3b+4c=0$, $5a+2b+c=0$

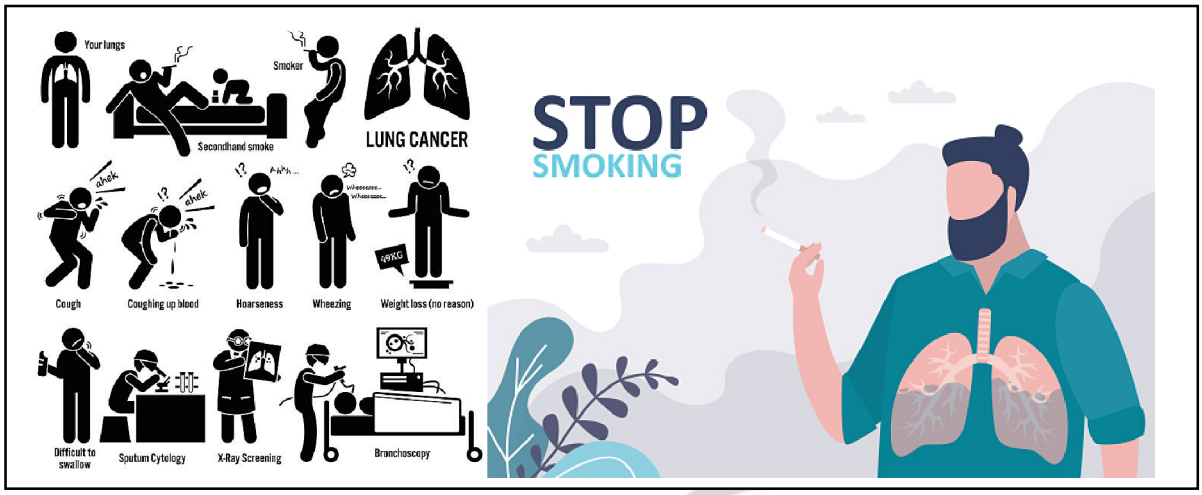
Therefore, $\frac{a}{3-8}=\frac{b}{20-2}=\frac{c}{4-15}$ i.e., $\frac{a}{-5}=\frac{b}{18}=\frac{c}{-11}$.

Hence, the required equation of line is $L:\frac{x+1}{-5}=\frac{y+1}{18}=\frac{z+1}{-11}$.

SECTION E

In this section there are 3 Case Study questions of 4 marks each.

36. Smoking increases the risk of lung problems.



A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung related problems. In a colony, 50 people were found to be smokers of which 30 are males.
A person is selected at random from these 50 people and tested for lung related problems.
Based on the given information, answer the following questions.
(i) What is the probability that selected person is a female?
(ii) If a male person is selected, what is the probability that he will not be suffering from lung problems?
(iii) (a) A person selected at random is detected with lung complications. Find the probability that selected person is a female.

OR

(iii) (b) A person selected at random is not having lung problems, find the probability that the person is a male.

Sol. Total smokers in the colony: 50, Number of males (M): 30.
So, the Number of females (F): $50-30=20$.

Also, the Probability a male smoker has lung complications is $P(L\,|\,M)=\dfrac{170}{1000}=0.17$.

And the Probability a female smoker has lung complications is $P(L\,|\,F)=\dfrac{120}{1000}=0.12$.

(i) $P(F)=\dfrac{20}{50}=\dfrac{2}{5}$.

(ii) $P(L'\,|\,M)=1-P(L\,|\,M)=1-\dfrac{17}{100}=\dfrac{83}{100}=0.83$.

REMARK If the question had been "What is the probability that a person selected at random is a male *and* does not have lung problems?", then the answer would be $P(M)\times P(L'\,|\,M)=\dfrac{30}{50}\times\dfrac{83}{100}=\dfrac{249}{500}=0.498$.

(iii) (a) Using Bayes' theorem, $P(F\,|\,L)=\dfrac{P(F).P(L\,|\,F)}{P(M).P(L\,|\,M)+P(F).P(L\,|\,F)}$

$$\Rightarrow P(F\,|\,L)=\frac{\frac{2}{5}.\frac{12}{100}}{\frac{30}{50}.\frac{17}{100}+\frac{2}{5}.\frac{12}{100}}=\frac{2\times12}{3\times17+2\times12}=\frac{24}{51+24}=\frac{24}{75}=\frac{8}{25}.$$

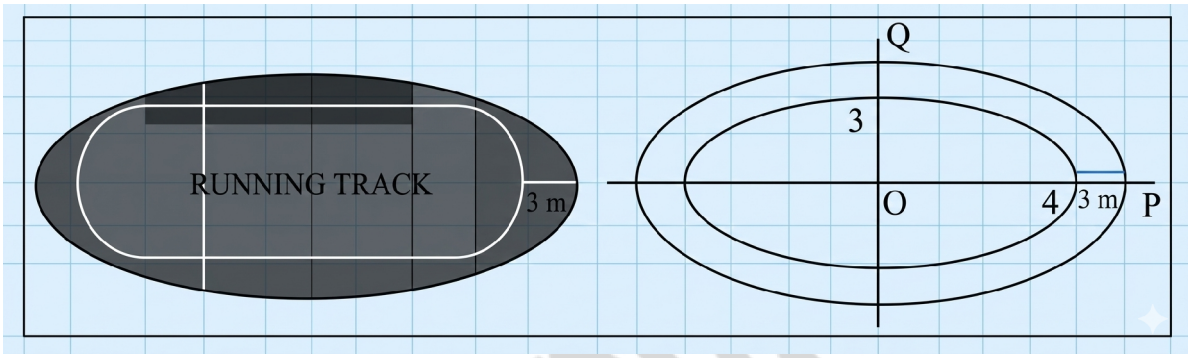
OR

(iii) (b) Note that, $P(L'\,|\,M)=0.83$, $P(L'\,|\,F)=1-0.12=0.88$.

Using Bayes' theorem, $P(M \mid L') = \dfrac{P(M).P(L' \mid M)}{P(M).P(L' \mid M) + P(F).P(L' \mid F)}$

$$\Rightarrow P(M \mid L') = \frac{\frac{30}{50}.\frac{83}{100}}{\frac{30}{50}.\frac{83}{100} + \frac{20}{50}.\frac{88}{100}} = \frac{30 \times 83}{30 \times 83 + 20 \times 88} = \frac{249}{249 + 176} = \frac{249}{425}.$$

37. A racing track is build around an elliptical ground whose equation is given by $9x^2 + 16y^2 = 144$. The width of the track is 3 m as shown below.



Based on the given information, answer the following questions.

(i) Express y as a function of x from the given equation of ellipse.

(ii) Integrate the function obtained in (i) with respect to x.

(iii) (a) Find the area of the region enclosed within the elliptical ground excluding the track using integration.

OR

(iii) (b) Write the coordinates of the points P and Q where the outer edge of the track cuts x axis and y axis in first quadrant and find the area of the triangle formed by points P, O, Q using integration.

Sol. **(i)** Given $9x^2 + 16y^2 = 144$ so, $16y^2 = 144 - 9x^2 \quad \Rightarrow y^2 = \frac{1}{16}(144 - 9x^2)$

$$\Rightarrow y = \frac{1}{4}\sqrt{144 - 9x^2}$$

$$\therefore y = \frac{3}{4}\sqrt{16 - x^2}.$$

(ii) Now $\int y\,dx = \int \frac{3}{4}\sqrt{16 - x^2}\,dx = \frac{3}{4}\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right] + C$

$$\therefore \int y\,dx = \frac{3x}{8}\sqrt{16 - x^2} + 6\sin^{-1}\frac{x}{4} + C$$

(iii) (a) Required area $= 4 \times \int_0^4 \frac{3}{4}\sqrt{16 - x^2}\,dx$

$$= 3\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$$

$$= 3\left\{\left[0 + 8\sin^{-1}1\right] - \left[0 + 8\sin^{-1}0\right]\right\}$$

$$= 3\left\{8 \times \frac{\pi}{2} - 0\right\} = 12\pi \text{ Sq. units}.$$

OR

(iii) (b) The track has a width of 3 m. Since the inner ellipse has $a = 4,\ b = 3$.

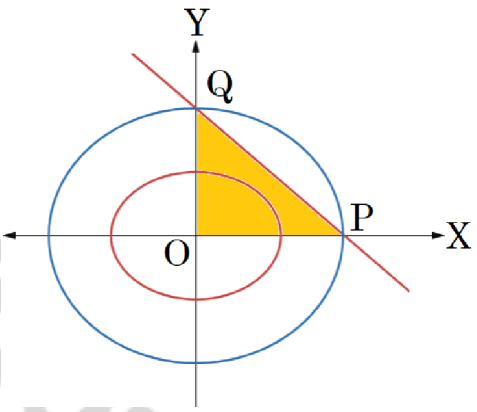
• Point P (on x-axis): The inner boundary is at $x = 4$.

Adding the track width (3 m), the outer boundary is at $4 + 3 = 7$.

So, P(7, 0).

• Point Q (on y-axis): The inner boundary is at $y = 3$. Adding the track width (3 m), the outer boundary is at $3 + 3 = 6$.

So, Q(0, 6).



The equation of line passing through P and Q is $\frac{x}{7} + \frac{y}{6} = 1$

$\Rightarrow y = 6\left(1 - \frac{x}{7}\right)$.

$$\text{Required area} = \int_0^7 y\,dx = \int_0^7 6\left(1 - \frac{x}{7}\right)dx$$

$$= \frac{6}{7}\left[\frac{(7-x)^2}{2(-1)}\right]_0^7$$

$$= -\frac{3}{7}[0 - 49] = -\frac{3}{7}(-49) = 21 \text{ Sq. units}.$$

38. Sports car racing is a form of motorsport which uses sports car prototypes. The competition is held on special tracks designed in various shapes.



The equation of one such track is given as follows:

$$f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \le x < 3 \\ x^2 + 40, & x \ge 3 \end{cases}$$

Based on the given, answer the following questions.

(i) Find $f'(x)$ for $0 < x < 3$.

(ii) Find $f'(4)$.

(iii) (a) Test for continuity of $f(x)$ at $x = 3$.

OR

(iii) (b) Test for differentiability of $f(x)$ at $x = 3$.

Sol. **(i)** For $0 < x < 3,\ f(x) = x^4 - 4x^2 + 4 \ \therefore f'(x) = 4x^3 - 8x$.

(ii) For $x \ge 3,\ f(x) = x^2 + 40 \ \therefore f'(x) = 2x$

Hence, $f'(4) = 2 \times 4 = 8$.

(iii) (a) Here $f(3) = 3^2 + 40 = 9 + 40 = 49$.

LHL : $\lim\limits_{x \to 3^-}(x^4 - 4x^2 + 4) = 3^4 - 4 \times 3^2 + 4 = 81 - 36 + 4 = 85 - 36 = 49$.

RHL : $\lim\limits_{x \to 3^+}(x^2 + 40) = 3^2 + 40 = 9 + 40 = 49$.

Since $f(3)=\lim\limits_{x\to3^-}f(x)=\lim\limits_{x\to3^+}f(x)$. So, $f(x)$ is continuous at $x=3$.

OR

(iii) (b) Here $f(3)=3^2+40=9+40=49$.

LHD : $\lim\limits_{x\to3^-}\dfrac{f(x)-f(3)}{x-3}=\lim\limits_{x\to3^-}\dfrac{x^4-4x^2+4-49}{x-3}=\lim\limits_{x\to3^-}\dfrac{x^4-4x^2-45}{x-3}=\lim\limits_{x\to3^-}\dfrac{(x^2-9)(x^2+5)}{x-3}$

$=\lim\limits_{x\to3^-}\dfrac{(x-3)(x+3)(x^2+5)}{x-3}=\lim\limits_{x\to3^-}\dfrac{(x+3)(x^2+5)}{1}=(3+3)(3^2+5)=84.$

RHD : $\lim\limits_{x\to3^+}\dfrac{f(x)-f(3)}{x-3}=\lim\limits_{x\to3^+}\dfrac{x^2+40-49}{x-3}=\lim\limits_{x\to3^+}\dfrac{x^2-9}{x-3}=\lim\limits_{x\to3^+}\dfrac{(x-3)(x+3)}{x-3}$

$=\lim\limits_{x\to3^+}\dfrac{(x+3)}{1}=3+3=6.$

Since LHD $\neq$ RHD. So, $f(x)$ is not differentiable at $x=3$.

☑ Q.P. Code - 65/2/2

01. $\int\dfrac{dx}{1+\cos x}$ is equal to

(a) $\dfrac{1}{2}\tan\dfrac{x}{2}+C$ (b) $\dfrac{-1}{2}\cot\dfrac{x}{2}+C$ (c) $-\cot\dfrac{x}{2}+C$ (d) $\tan\dfrac{x}{2}+C$

Sol. (d) $\int\dfrac{dx}{1+\cos x}=\int\dfrac{dx}{2\cos^2\dfrac{x}{2}}=\dfrac{1}{2}\int\sec^2\dfrac{x}{2}dx=\dfrac{1}{2}\left(\dfrac{\tan\dfrac{x}{2}}{\dfrac{1}{2}}\right)+C=\tan\dfrac{x}{2}+C.$

04. The general solution of the differential equation $xdy-ydx=0$ is

(a) $x^2-y^2=k$ (b) $xy=k$ (c) $x=ky$ (d) $\log y+\log x=k$

Sol. (c) Rewriting the D.E., $\dfrac{dy}{y}-\dfrac{dx}{x}=0$

$\Rightarrow\int\dfrac{dy}{y}-\int\dfrac{dx}{x}=0$

$\Rightarrow\log|y|-\log|x|=\log|C|$

$\Rightarrow\log\left|\dfrac{y}{x}\right|=\log|C|$

$\Rightarrow\dfrac{y}{x}=C$ or, $x=ky$ where $k=\dfrac{1}{C}$.

08. Direction cosines of line $x=y=1-z$ are

(a) 1, 1, 1 (b) $\dfrac{1}{\sqrt3},\dfrac{1}{\sqrt3},\dfrac{-1}{\sqrt3}$ (c) 0, 0,1 (d) $\dfrac{1}{\sqrt3},\dfrac{1}{\sqrt3},\dfrac{1}{\sqrt3}$

Sol. (b) Rewriting the given line, we get $\dfrac{x}{1}=\dfrac{y}{1}=\dfrac{z-1}{-1}$. So, its d.r.'s are1, 1, –1.

The d.c.'s will be $\dfrac{1}{\sqrt{1^2+1^2+(-1)^2}}.\dfrac{1}{\sqrt{1+1+1}},\dfrac{-1}{\sqrt3}$ i.e., $\dfrac{1}{\sqrt3},\dfrac{1}{\sqrt3},\dfrac{-1}{\sqrt3}$.

12. A relation R on set $A=\{1,2,3\}$ defined as $R=\{(1,2),(2,1),(2,2)\}$ is

(a) Reflexive only (b) Reflexive and Transitive

(c) Symmetric an Transitive (d) Symmetric only

Sol. (d) Symmetric only.
Note that, $(1,1) \notin R$, $(3,3) \notin R$. So, R can't be reflexive.
Also $(1,2) \in R$, $(2,1) \in R$ but $(1,1) \notin R$. So, R can't be transitive.

15. If $A = \begin{bmatrix} \frac{1}{2}\cos x & -\sin x \\ \sin x & \frac{1}{2}\cos x \end{bmatrix}$ and $A + A' = I$, then value of $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) 0 (d) $\frac{-\pi}{2}$

Sol. (c) $\because A + A' = I \Rightarrow \begin{bmatrix} \frac{1}{2}\cos x & -\sin x \\ \sin x & \frac{1}{2}\cos x \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\cos x & \sin x \\ -\sin x & \frac{1}{2}\cos x \end{bmatrix} = I$

$\Rightarrow \begin{bmatrix} \cos x & 0 \\ 0 & \cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By the def. of equality of matrices, we get $\cos x = 1 \ \therefore x = 0 \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

18. If $e^{x+y} = 3x$, then $\frac{dy}{dx}$ is

(a) $\frac{3}{e^{x+y}}$ (b) $\frac{1}{e^{x+y}}$ (c) $\frac{1-e^{x+y}}{e^{x+y}}$ (d) $\frac{3-e^{x+y}}{e^{x+y}}$

Sol. (d) On differentiating w.r.t. x both the sides, $e^{x+y}\left(1+\frac{dy}{dx}\right) = 3$

$\Rightarrow \left(1+\frac{dy}{dx}\right) = \frac{3}{e^{x+y}}$

$\Rightarrow \frac{dy}{dx} = \frac{3}{e^{x+y}} - 1$

$\therefore \frac{dy}{dx} = \frac{3-e^{x+y}}{e^{x+y}}$.

23. Find the coordinates of foot of perpendicular drawn from (0, 0, 0) to line $\frac{x}{1} = \frac{y+1}{-1} = \frac{z-3}{-2}$.

Sol. Equation of line : $\frac{x}{1} = \frac{y+1}{-1} = \frac{z-3}{-2} = \lambda$ *say.*

$\therefore$ Coordinates of any random point on this line is $P(\lambda, -\lambda-1, -2\lambda+3)$.

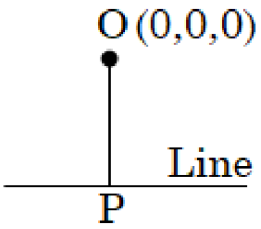
The direction ratios of OP are $\lambda, -\lambda-1, -2\lambda+3$.

As AP $\perp$ line so, $1(\lambda) + (-1)(-\lambda-1) + (-2)(-2\lambda+3) = 0$

$\Rightarrow \lambda + \lambda + 1 + 4\lambda - 6 = 0$

$\Rightarrow 6\lambda = 5$

$\Rightarrow \lambda = \frac{5}{6}$



$\therefore P\left(\frac{5}{6}, -\frac{11}{6}, \frac{8}{6}\right)$ or $P\left(\frac{5}{6}, -\frac{11}{6}, \frac{4}{3}\right)$ is the required foot of perpendicular.

25. If $x = \sin t - \cos t$, $y = \sin t \cos t$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

Sol. Given $x = \sin t - \cos t$, $y = \sin t \cos t$

Differentiating w.r.t. t, we get $\frac{dx}{dt} = \cos t + \sin t$, $\frac{dy}{dt} = \sin t(-\sin t) + \cos t(\cos t)$

As $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ so, $\frac{dy}{dx} = \frac{-\sin^2 t + \cos^2 t}{\cos t + \sin t}$

$\Rightarrow \frac{dy}{dx} = \frac{(\cos t + \sin t)(\cos t - \sin t)}{\cos t + \sin t} = (\cos t - \sin t)$

Hence, $\frac{dy}{dx}\left(\text{at } t = \frac{\pi}{4}\right) = \cos\frac{\pi}{4} - \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$.

REMARK Since the question does not ask to find $\frac{dy}{dx}$ specifically, so we can skip finding it in the simplified form and replace the value of t after getting $\frac{dy}{dx}$ in the first place itself.

28. Solve the following linear programming problem graphically.

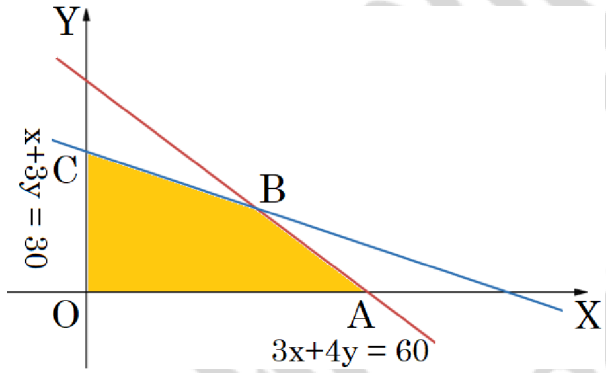
Maximize $Z = 8000x + 12000y$

Subject to constraints $3x + 4y \le 60$, $x + 3y \le 30$, $x \ge 0$, $y \ge 0$.

Sol. Consider the graph shown.

The corner points are O(0, 0), A(20, 0), B(12, 6) and C(0, 10).

Therefore, $Z_O = 0$, $Z_A = 160000$, $Z_B = 168000$, $Z_C = 120000$.



Hence, the maximum value of Z is 168000 at the corner point (12, 6).

30. Find $\int \frac{2x+1}{\sqrt{6x+x^2}} dx$.

Sol. Let $I = \int \frac{2x+1}{\sqrt{6x+x^2}} dx$

$\Rightarrow I = \int \frac{(2x+6)-5}{\sqrt{6x+x^2}} dx$

$\Rightarrow I = \int \frac{(2x+6)}{\sqrt{6x+x^2}} dx - \int \frac{5}{\sqrt{6x+x^2}} dx$ $\left[\because \int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + C\right.$

$\Rightarrow I = 2\sqrt{6x+x^2} - \int \frac{5}{\sqrt{(x+3)^2 - 3^2}} dx$

$\Rightarrow I = 2\sqrt{6x+x^2} - 5\log\left|(x+3) + \sqrt{(x+3)^2 - 3^2}\right| + C$

$\therefore I = 2\sqrt{6x+x^2} - 5\log\left|(x+3) + \sqrt{6x+x^2}\right| + C$.

32. Find the domain of $p(x)=\sin^{-1}(1-2x^2)$. Hence, find the value of x for which $p(x)=\frac{\pi}{6}$. Also, write the range of $2p(x)+\frac{\pi}{2}$.

Sol. Note that, the function $p(x)$ is defined when $-1\le(1-2x^2)\le1$

$\Rightarrow -1-1\le(1-2x^2)-1\le1-1$

$\Rightarrow -2\le-2x^2\le0$

$\Rightarrow 1\ge x^2\ge0$

$\Rightarrow -1\le x\le1$

$\therefore$ Domain of $p(x)$ is $x\in[-1,1]$.

Now taking $p(x)=\frac{\pi}{6}$

$\Rightarrow \sin^{-1}(1-2x^2)=\frac{\pi}{6}$

$\Rightarrow \sin[\sin^{-1}(1-2x^2)]=\sin\frac{\pi}{6}$

$\Rightarrow (1-2x^2)=\frac{1}{2}$

$\Rightarrow -2x^2=\frac{1}{2}-1$

$\Rightarrow -2x^2=-\frac{1}{2}$

$\Rightarrow x^2=\frac{1}{4}$

$\therefore x=\pm\frac{1}{2}$

Since range of $p(x)=\sin^{-1}(1-2x^2)$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ for all $x\in[-1,1]$.

That is, $-\frac{\pi}{2}\le p(x)\le\frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2}\times2\le2p(x)\le\frac{\pi}{2}\times2$

$\Rightarrow -\pi+\frac{\pi}{2}\le2p(x)+\frac{\pi}{2}\le\pi+\frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2}\le2p(x)+\frac{\pi}{2}\le\frac{3\pi}{2}$

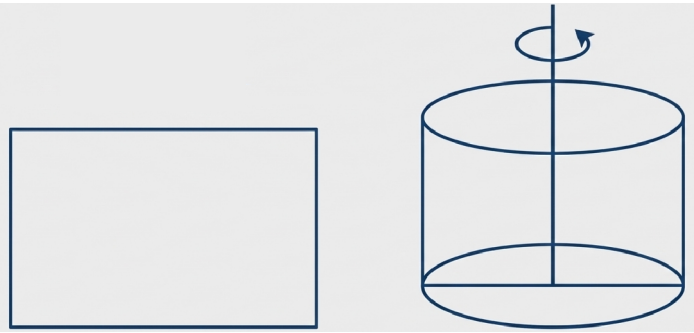
Hence, the required range of $2p(x)+\frac{\pi}{2}$ is $\left[-\frac{\pi}{2},\frac{3\pi}{2}\right]$.

35. (a) Find the sub intervals of $(0,\pi)$ in which $f(x)=\tan^{-1}(\sin x-\cos x)$ is increasing and decreasing.

OR

(b) A rectangle of perimeter 24 cm is revolved along one of its sides to sweep out a cylinder of maximum volume. Find the dimensions of the rectangle.

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Sol. (a) For $f(x) = \tan^{-1}(\sin x - \cos x),\ x \in (0, \pi)$

$$\Rightarrow f'(x) = \frac{1}{1+(\sin x - \cos x)^2} \times (\cos x + \sin x)$$

$$\Rightarrow f'(x) = \frac{(\cos x + \sin x)}{1+\sin^2 x + \cos^2 x - 2\sin x \cos x} = \frac{(\cos x + \sin x)}{2 - \sin 2x}$$

For $f'(x) = 0$, $\frac{(\cos x + \sin x)}{2 - \sin 2x} = 0 \quad \Rightarrow \sin x = -\cos x$

$$\Rightarrow \tan x = -1$$

$$\therefore x = \frac{3\pi}{4} \in (0, \pi)$$

For $x \in \left(0, \frac{3\pi}{4}\right]$, $f'(x) \geq 0$ so, f is increasing in this interval.

For $x \in \left[\frac{3\pi}{4}, \pi\right)$, $f'(x) \leq 0$ so, f is decreasing in this interval.

OR

(b) Suppose the rectangle is rotated about its length. Then the length L of rectangle will be height h of the cylinder thus generated. Also the width b of rectangle will be acting as the radius of the base of cylinder.

As perimeter of the rectangle $= 2(L + b) = 24 \quad \Rightarrow L = 12 - b$

Now volume of the cylinder is, $V = \pi r^2 h$

$$\Rightarrow V = \pi b^2 L$$

$$\Rightarrow V = \pi b^2 (12 - b)$$

$$\Rightarrow V = \pi(12b^2 - b^3)$$

$$\Rightarrow \frac{dV}{db} = \pi(24b - 3b^2)$$

$$\Rightarrow \frac{d^2V}{db^2} = \pi(24 - 6b)$$

For local points of maxima and/or minima, $\frac{dV}{db} = \pi(24b - 3b^2) = 0$

$$\Rightarrow 3\pi b(8 - b) = 0$$

$\because b \neq 0 \ \therefore b = 8$ cm, $L = 12 - 8 = 4$ cm

$$\because \left.\frac{d^2V}{db^2}\right]_{\text{at } b=8 \text{ cm}} = \pi(24 - 48) = -24\pi < 0$$, so V is maximum at $b = 8$ cm.

Hence, the dimensions of rectangle are $8 \text{ cm} \times 4 \text{ cm}$.

☑ Q.P. Code - 65/2/3

01. Direction cosines of line $\frac{1-x}{0} = y = z$ are

(a) 1, 1, 1 (b) $0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$ (c) 1, 0, 0 (d) 0, –1, –1

Sol. (b) Rewriting the line, we get $\frac{x-1}{0} = \frac{y}{1} = \frac{z}{1}$. So, its d.r.'s are 0, 1, 1.

Then its d.c.'s will be $\frac{0}{\sqrt{0^2+1^2+1^2}}, \frac{1}{\sqrt{0+1+1}}, \frac{1}{\sqrt{2}}$ i.e., $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$.

Since direction cosines can also be represented by their negatives (on reversing the direction of the line), we can also write them as $0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$.

05. A relation R on set $A = \{1,2,3\}$ is defined as $R = \{(1,3),(3,3),(1,1),(2,2),(3,1)\}$ is

(a) only reflexive and symmetric
(b) reflexive only
(c) only reflexive and transitive
(d) reflexive, symmetric and transitive

Sol. (d) reflexive, symmetric and transitive.

08. If $A = \begin{bmatrix} \tan x & \cot x \\ -\cot x & \tan x \end{bmatrix}$ and $A + A' = 2I$, then value of $x \in \left[0, \frac{\pi}{2}\right]$ is

(a) 0 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

Sol. (c) As $A + A' = 2I$ so, $\begin{bmatrix} \tan x & \cot x \\ -\cot x & \tan x \end{bmatrix} + \begin{bmatrix} \tan x & -\cot x \\ \cot x & \tan x \end{bmatrix} = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2\tan x & 0 \\ 0 & 2\tan x \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

By the def. of equality of matrices, we get $2\tan x = 2$

$\Rightarrow \tan x = 1$

$\therefore x = \frac{\pi}{4} \in \left[0, \frac{\pi}{2}\right]$.

11. If $x + y = xy$, then $\frac{dy}{dx}$ is

(a) $\frac{y}{x-1}$ (b) $\frac{1}{x-1}$ (c) $\frac{y-1}{x-1}$ (d) $\frac{1-y}{x-1}$

Sol. (d) On differentiating w.r.t. x both the sides, we get $1 + \frac{dy}{dx} = x\frac{dy}{dx} + y \times 1$

$\Rightarrow \frac{dy}{dx} - x\frac{dy}{dx} = y - 1$

$\Rightarrow \frac{dy}{dx}(1-x) = y - 1$

$\Rightarrow \frac{dy}{dx} = \frac{y-1}{(1-x)}$

$\therefore \frac{dy}{dx} = \frac{1-y}{x-1}$.

12. $\int \frac{dx}{\sec x + \tan x}$ is equal to

(a) $\log|\sec x + \tan x| + C$ (b) $\log|\sec x - \tan x| + C$

(c) $\log|1 + \cos x| + C$ (d) $\log|1 + \sin x| + C$

Sol. (d) $\int \frac{dx}{\sec x + \tan x} = \int \frac{\cos x\, dx}{1 + \sin x} = \log|1 + \sin x| + C$ $\left[\because \int \frac{f'(x)\,dx}{f(x)} = \log|f(x)| + C\right.$

15. The general solution of the differential equation $x^2dy + y^2dx = 0$ is

(a) $x^3 + y^3 = k$ (b) $\frac{1}{y} - \frac{1}{x} = k$ (c) $\frac{1}{y} + \frac{1}{x} = k$ (d) $\log y^2 + \log x^2 = k$

Sol. (c) Rewriting the D.E., we get $\frac{dy}{y^2} + \frac{dx}{x^2} = 0$

$\Rightarrow \int \frac{dy}{y^2} + \int \frac{dx}{x^2} = 0$

$\Rightarrow -\frac{1}{y} - \frac{1}{x} = C$

Therefore, $\frac{1}{y} + \frac{1}{x} = k$, where $k = -C$.

21. Find the coordinates of the point on line $x = \frac{y-1}{2} = \frac{z-2}{3}$ whose y coordinate is 3 times the x coordinate.

Sol. Let $x = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$.

The coordinates of any random point on the line is $P(\lambda, 2\lambda + 1, 3\lambda + 2)$.

Since y coordinate is 3 times the x coordinate so, $2\lambda + 1 = 3\lambda \Rightarrow \lambda = 1$

Hence, the required point is : P(1, 3, 5).

23. If $x = e^{\sin^{-1} t}$, $y = e^{\cos^{-1} t}$, then find $\frac{dy}{dx}$ at $t = \frac{1}{\sqrt{2}}$.

Sol. $x = e^{\sin^{-1} t}$, $y = e^{\cos^{-1} t}$

Differentiating w.r.t. t, we get $\frac{dx}{dt} = e^{\sin^{-1} t} \times \frac{1}{\sqrt{1-t^2}}$, $\frac{dy}{dt} = e^{\cos^{-1} t} \times \left(-\frac{1}{\sqrt{1-t^2}}\right)$

As $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ so, $\frac{dy}{dx} = e^{\cos^{-1} t} \times \left(-\frac{1}{\sqrt{1-t^2}}\right) \times \frac{\sqrt{1-t^2}}{e^{\sin^{-1} t}}$

$\Rightarrow \frac{dy}{dx} = -e^{\cos^{-1} t - \sin^{-1} t}$

At $t = \frac{1}{\sqrt{2}}$, $\frac{dy}{dx} = -e^{\cos^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}\frac{1}{\sqrt{2}}} = -e^{\frac{\pi}{4} - \frac{\pi}{4}} = -e^0 = -1$.

27. Find $\int \frac{3x-1}{\sqrt{x^2 - 4x}} dx$.

Sol. Let $I = \int \frac{3x-1}{\sqrt{x^2 - 4x}} dx$

$$\Rightarrow I = 3\int \frac{x-\frac{1}{3}}{\sqrt{x^2-4x}}dx$$

$$\Rightarrow I = \frac{3}{2}\int \frac{(2x-4)+4-\frac{2}{3}}{\sqrt{x^2-4x}}dx$$

$$\Rightarrow I = \frac{3}{2}\int \frac{(2x-4)+\frac{10}{3}}{\sqrt{x^2-4x}}dx$$

$$\Rightarrow I = \frac{3}{2}\left[\int \frac{(2x-4)}{\sqrt{x^2-4x}}dx + \frac{10}{3}\int \frac{1}{\sqrt{x^2-4x}}dx\right] \qquad \left[\because \int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)}+C\right.$$

$$\Rightarrow I = \frac{3}{2}\left[2\sqrt{x^2-4x} + \frac{10}{3}\int \frac{1}{\sqrt{(x-2)^2-2^2}}dx\right]$$

$$\Rightarrow I = 3\sqrt{x^2-4x} + 5\log\left|(x-2)+\sqrt{(x-2)^2-2^2}\right| + C$$

$$\therefore I = 3\sqrt{x^2-4x} + 5\log\left|(x-2)+\sqrt{x^2-4x}\right| + C\,.$$

31. Solve the following linear programming problem graphically.

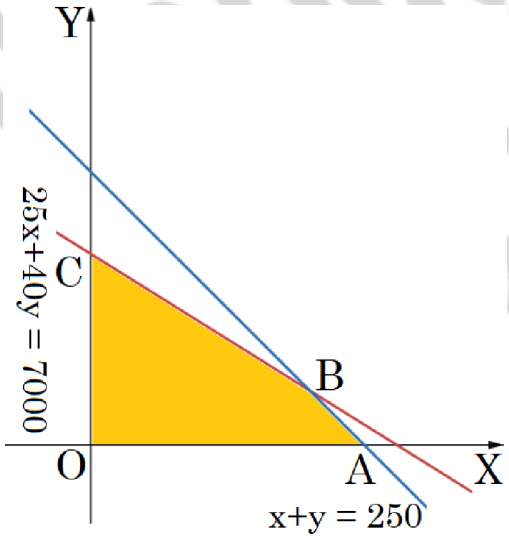
Maximize $Z = 4500x + 5000y$

Subject to the constraints $x + y \le 250,\ 25x + 40y \le 7000,\ x \ge 0,\ y \ge 0$.

Sol. Consider the graph shown below.

The corner points are O(0, 0), A(250, 0), B(200, 50) and C(0, 175).

Therefore, $Z_O = 0,\ Z_A = 1125000,\ Z_B = 1150000,\ Z_C = 875000$.

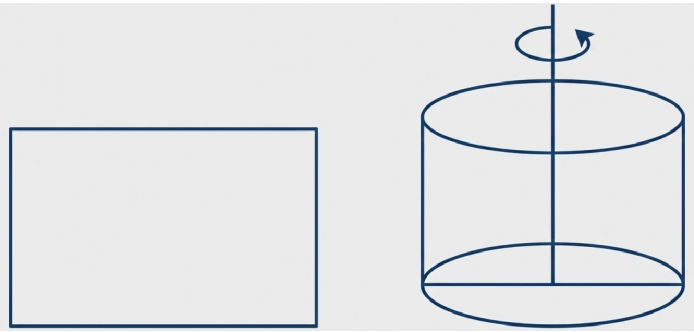


The maximum value of Z is 1150000, which occurs at the point (200, 50).

32. (a) Find the sub intervals of $\left(0, \frac{\pi}{2}\right)$ in which $f(x) = \log(\sin x + \cos x)$ is increasing and decreasing.

OR

(b) A rectangle of perimeter 30 cm is revolved along one of its sides to sweep out a cylinder of maximum volume. Find the dimensions of the rectangle.



Sol. (a) For $f(x)=\log(\sin x+\cos x)$, $x\in\left(0,\frac{\pi}{2}\right)$

$$\Rightarrow f'(x)=\frac{1}{(\sin x+\cos x)}\times(\cos x-\sin x)$$

$$\Rightarrow f'(x)=\frac{(\cos x-\sin x)}{(\sin x+\cos x)}$$

For $f'(x)=0$, $\frac{(\cos x-\sin x)}{(\sin x+\cos x)}=0 \quad \Rightarrow \cos x=\sin x$

$$\therefore x=\frac{\pi}{4}\in\left(0,\frac{\pi}{2}\right)$$

For $x\in\left(0,\frac{\pi}{4}\right]$, $f'(x)\geq 0$ so, f is increasing in this interval.

For $x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right)$, $f'(x)\leq 0$ so, f is decreasing in this interval.

OR

(b) Suppose the rectangle is rotated about its length. Then the length L of rectangle will be height h of the cylinder thus generated. Also the width b of rectangle will be acting as the radius of the base of cylinder.

As perimeter of the rectangle $=2(L+b)=30 \quad \Rightarrow L=15-b$

Now volume of the cylinder is, $V=\pi r^2h$

$$\Rightarrow V=\pi b^2L$$

$$\Rightarrow V=\pi b^2(15-b)$$

$$\Rightarrow V=\pi(15b^2-b^3)$$

$$\Rightarrow \frac{dV}{db}=\pi(30b-3b^2)$$

$$\Rightarrow \frac{d^2V}{db^2}=\pi(30-6b)$$

For local points of maxima and/or minima, $\frac{dV}{db}=\pi(30b-3b^2)=0$

$$\Rightarrow 3\pi b(10-b)=0$$

$\because b\neq 0 \ \therefore b=10$ cm, $L=15-10=5$ cm

$$\because \left.\frac{d^2V}{db^2}\right]_{\text{at } b=10\text{ cm}}=\pi(30-60)=-30\pi<0,$$ so V is maximum at $b=10$ cm.

Hence, the dimensions of rectangle are $10\text{ cm}\times 5\text{ cm}$.

33. Find the domain of $q(x)=\cos^{-1}(4x^2-3)$. Hence, find the value of x for which $q(x)=0$. Also, write the range of $3q(x)-\pi$.

Sol. Since $q(x)=\cos^{-1}(4x^2-3)$ is defined when $-1\le(4x^2-3)\le1$

$\Rightarrow -1+3\le(4x^2-3)+3\le1+3$

$\Rightarrow 2\le 4x^2\le 4$

$\Rightarrow \frac{1}{2}\le x^2\le 1$

Either $-1\le x\le-\frac{1}{\sqrt{2}}$ (for the positive side) or $\frac{1}{\sqrt{2}}\le x\le 1$ (for the negative side).

So, domain of $q(x)$ is $x\in\left[-1,-\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},1\right]$.

Taking $q(x)=0$, we get $\cos^{-1}(4x^2-3)=0$

$\Rightarrow \cos[\cos^{-1}(4x^2-3)]=\cos 0$

$\Rightarrow (4x^2-3)=1$

$\Rightarrow 4x^2=4 \quad \Rightarrow x^2=1$

$\therefore x=\pm1$

Since the range of $q(x)=\cos^{-1}(4x^2-3)$ is $[0,\pi]$ for all values of $x\in\left[-1,-\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},1\right]$.

So, $0\le q(x)\le\pi$

$\Rightarrow 0\times3\le 3q(x)\le 3\pi$

$\Rightarrow 0-\pi\le 3q(x)-\pi\le 3\pi-\pi$

$\Rightarrow -\pi\le 3q(x)-\pi\le 2\pi$

Hence, the range of $3q(x)-\pi$ is $[-\pi,2\pi]$.

With regards,
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✔ Quality Mathematics Resources
✔ Important Practice Questions & Updates
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✔ Healthy Academic Interaction

ⓘ How to Join?

Please **scan the QR-Code** corresponding to your category (Teachers / Class IX - X Students / Class XI - XII Students) to join the relevant group.
Alternatively, you can **touch the QR-Code** too, after opening in the Drive PDF App.

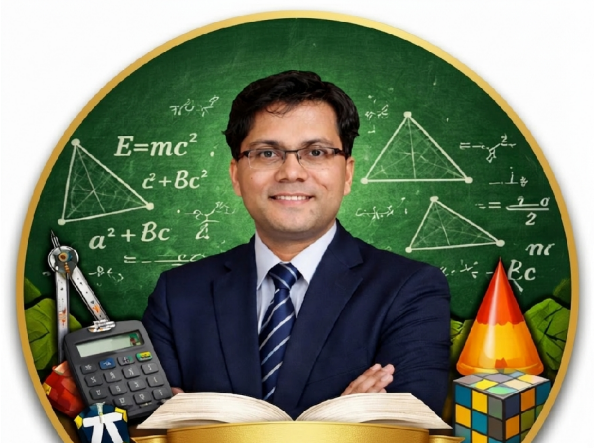
✪ Important Guidelines

- Teachers are requested NOT to join student groups.
- Students are requested NOT to join teachers' groups.

☑ If you are already a member of any of our existing groups, please avoid joining another group to prevent repeated notifications of the same resources.
Instead, you may share this opportunity with your colleagues or students who may benefit from these Mathematics learning communities.

With Regards
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Dedicated to helping students and teachers strengthen conceptual understanding and excel in Mathematics.